

Relaxing Nucleosynthesis Constraints on Brans-Dicke Theories

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We reconsider constraints on Brans-Dicke theories arising from the requirement of successful Big Bang Nucleosynthesis. Such constraints typically arise by imposing that the universe be radiation-dominated at early times, and therefore restricting the contribution that a Brans-Dicke scalar could make to the energy budget of the universe. However, in this paper we show how the dynamics of the Brans-Dicke scalar itself can mimic a radiation-dominated kinematics, thereby allowing successful nucleosynthesis with a sizable contribution to the total cosmic energy density. In other words Newton's constant may dynamically acquire values quite different from that today, even though the evolution mimics radiation domination. This possibility significantly relaxes the existing bounds on Brans-Dicke fields, and opens the door to new possibilities for early universe cosmology. The necessary fine tunings required by such an arrangement are identified and discussed.

*Dedicated to Rafael Sorkin, on the occasion of his 60th birthday,
and to celebrate his wonderful contributions to physics*

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I. INTRODUCTION

In the last few years, there have been a large number of different approaches to the dark energy enigma. As is well-known, the anisotropies of the Cosmic Microwave Background (CMB) radiation [1] and data from type Ia Supernovae [2, 3, 4], are well-fit by a cosmological constant, albeit a fine-tuned one, of which we have no satisfactory theoretical understanding. Another possibility is that the cosmological constant is precisely zero (or at most subdominant) and that a dynamical component is driving cosmic acceleration. This component may be a manifestation of a large-scale modification of General Relativity [5, 6, 7, 8, 9, 10, 11, 12], or a new dynamical field, such as a scalar field [13, 14, 15, 16, 17]. It is quite natural to consider scalar fields that are non-minimally coupled to gravity [18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35]. Such fields arise both in the context of string theory, and in more general theories with extra spatial dimensions. These fields are typically of the Brans-Dicke (BD) type, and have been considered in a large variety of other cosmological contexts, such as inflation and baryogenesis [36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47].

However, there exist tight constraints on BD fields. Some of these constraints are on solar system scales, constraining the BD parameter to satisfy $\omega > 40000$. Other constraints arise from cosmology itself. For example, in order not to spoil the success of the Big Bang Nucleosynthesis (BBN) predictions, the scalar field component should be subdominant to radiation and the scale factor must evolve nearly as $t^{1/2}$. This constraint is quite well accepted, and a detailed analysis of the constraints on other kinds of evolutions, such as dark matter dominated one, can be found in [48].

In this paper we revisit this particular issue, considering whether a BD scalar field might play a *significant* role in cosmic evolution during BBN, while maintaining an acceptable evolution of the scale factor. We shall only require the BD field to drive the kinematics of the universe, making it expand *as if* radiation dominates. It should be emphasized that in our model the expansion rate (Hubble function H) is driven by the BD field, while the scattering rates are determined by cross sections and abundances of the standard fields of the plasma (e^\pm , neutrinos, photons, baryons). One consequence is that the relation between temperature and time becomes an adjustable parameter in this model. Provided the BD field can reproduce a standard kinematics $H(a)$, one is free e.g. to normalize the abundances of all the fields to the value they assume at the same $H(a)$ in the standard

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model. Since they do not interact with the BD field but gravitationally, the particles in the plasma cannot distinguish which component is driving the evolution of the universe, and they will interact among themselves in the same way as if the BD field is not present. As a consequence, the production rate for the light elements is the same and the freeze-out temperatures for the reactions are unchanged.

The big difference between this model and other BD models during BBN (see e.g. [49]), is that now the BD field, i.e Newton's constant, can be largely different from today's value at the time of BBN. This is quite different from a standard approach in which the BD field is allowed to be just slightly different from today's value. However, just after BBN, some non-trivial dynamics is required to bring the BD fields to values consistent with a standard evolution for the universe. This is sufficient to ensure that the temperature of the universe has the usual behavior from the end of BBN up to now. We shall see that such a scenario is possible, while remaining consistent with solar system constraints. However, as one might expect, this interesting possibility can only arise under finely tuned conditions. We describe the tuning required, both on the initial conditions of the scalar and on its associated potential.

II. SCALAR TENSOR THEORIES

We consider the BD Lagrangian density with a potential $W(\varphi)$ as

$$\mathcal{L} = \sqrt{-g} \left[\varphi R - \frac{\omega}{\varphi} (\partial_\mu \varphi)^2 - W(\varphi) + 16\pi \mathcal{L}_m \right] , \quad (1)$$

where R is the Ricci scalar, φ is a real scalar field with units of $[\text{mass}]^2$, ω is the BD parameter and \mathcal{L}_m denotes the matter Lagrangian density. Our signature is $-+++$. Typically, as long as the potential is too weak to confine the scalar field, precision measurements of the timing of signals from the Cassini mission yield the bound $\omega > 40000$ [50]. Although it has been suggested that this bound may not hold on cosmological scales [51], we will take a conservative view and demand that it be satisfied.

Varying the action with respect to the metric tensor gives

$$G_{\mu\nu} - \frac{\omega}{\varphi^2} \partial_\mu \varphi \partial_\nu \varphi - \frac{1}{\varphi} \nabla_\mu \nabla_\nu \varphi + g_{\mu\nu} \left[\frac{\square \varphi}{\varphi} + \frac{\omega}{2\varphi^2} (\partial\varphi)^2 + \frac{W}{2\varphi} \right] = \frac{8\pi}{\varphi} T_{\mu\nu} , \quad (2)$$

and combining this result with the equation obtained by varying the action with respect to φ one obtains

$$\square \varphi = \frac{8\pi}{2\omega + 3} T + \frac{1}{2\omega + 3} [\varphi W_{,\varphi} - 2W] , \quad (3)$$

where $T = T_\mu{}^\mu$ and $W_{,\varphi}$ denotes $dW/d\varphi$.

To study the cosmological aspects of this theory in the framework of a homogeneous and isotropic universe, we use the Friedmann, Robertson-Walker (FRW) ansatz for the metric

$$ds^2 = a^2(\eta) (-d\eta^2 + d\mathbf{x}^2) , \quad (4)$$

where $a(\eta)$ is the scale factor and η is conformal time, related to the usual cosmic time through $a(\eta)d\eta = dt$. We have assumed a zero curvature contribution, consistent with current data.

If we introduce the dimensionless field $\phi \equiv \varphi/M_P^2$ — M_P being the Planck mass—the equations of motion become

$$\ddot{\phi} = -2\frac{\dot{a}}{a}\dot{\phi} + \frac{8\pi a^2}{(2\omega + 3)M_P^2} \left[\rho - 3P + \frac{W}{4\pi} - \frac{\phi}{8\pi} \frac{dW}{d\phi} \right] , \quad (5)$$

$$\frac{\dot{a}}{a} = -\frac{\dot{\phi}}{2\phi} + \left[\frac{2\omega + 3}{12} \left(\frac{\dot{\phi}}{\phi} \right)^2 + \frac{8\pi a^2}{3M_P^2 \phi} \left(\rho + \frac{W}{16\pi} \right) \right]^{1/2} , \quad (6)$$

where ρ and P refer to the total energy density and pressure of the standard plasma components respectively, and a dot denotes a derivative with respect to conformal time η .

III. MIMICKING RADIATION DURING BIG BANG NUCLEOSYNTHESIS

In the standard scenario, the universe is radiation dominated during BBN. This requirement leads to theoretical expectations that are in reasonable agreement with observations, at least at the 2σ level. In particular

the Deuterium number density [53] points to a value for the baryon content of the universe which is within $1\text{-}\sigma$ equal to the independent determination of this parameter extracted from the CMB power spectrum [1]

$$\Omega_b h^2 = 0.0223 \pm 0.0008 \quad . \quad (7)$$

This result is compatible with a radiation content given by photons and three weakly interacting neutrinos/antineutrinos (for a review of the present status of standard BBN see e.g. [54, 55, 56]).

Nevertheless, present data still leave room for a non-standard energy density content, or a non standard value for the Hubble expansion rate. For example, the analysis carried out in [55] shows that a conservative observational range for the helium-4 mass fraction constrains any extra contribution or deficit in the radiation energy content in the range $-1.1 \lesssim \Delta N_{\text{eff}} \lesssim 0.8$, when parameterizing it in terms of extra effective neutrino species.

In view of these considerations, the main question we would like to address is whether it is still possible that the BD field ϕ contributes during BBN in a non negligible way. The effect of ϕ can be easily discerned from Eq. (6). In particular, its dynamics affects the value of the gravitational constant which scales as $1/\phi$ and which ties the energy-density to the expansion rate. Moreover, the dynamics of ϕ provides extra contributions to the Hubble parameter. The usual assumption is to consider a negligible role of the functions $W/16\pi\phi$ (the “potential energy term”) and $K \equiv (2\omega + 3)M_P^2(\dot{\phi}/\phi)^2/32\pi a^2$ (the “kinetic energy term”) in Eq. (6)¹, so that the usual BBN scenario is only possibly changed by a different value of the effective gravitational constant or, equivalently, of a re-scaled energy density, see e.g. [57]. The same bound previously quoted in term of N_{eff} then translates into $0.9 \lesssim \phi \lesssim 1.2$.

We now relax the hypothesis that the ϕ field dynamics contributes negligibly at the epoch of BBN. Instead, we consider the possibility that the behavior of ϕ during the temperature interval $1 \text{ MeV} \geq T \geq 0.01 \text{ MeV}$ may be highly nontrivial and yet may still result in primordial abundances for ${}^2\text{H}$, ${}^3\text{He}$, ${}^4\text{He}$ and ${}^7\text{Li}$ of the correct order of magnitude. In particular, we consider a specific, yet interesting case, in which the universe during this stage is still *radiation* dominated, but by this we now only mean that the specific dependence of the scale factor $a(t) \propto t^{1/2}$, or equivalently in terms of the conformal time

$$a(\eta) = \kappa \eta \quad , \quad (8)$$

where κ is a constant to be evaluated by cosmological observables. The condition (8) in turn imposes severe constraints on Eqs. (5) and (6), which under this ansatz can be recast into

$$\phi'' = -\frac{2}{\eta}\phi' + \frac{8\pi a^2}{\kappa^2 M_P^2 (2\omega + 3)} \left[\rho - 3P + \frac{W}{4\pi} - \frac{\phi}{\phi'} \frac{W'}{8\pi} \right] \quad , \quad (9)$$

$$\frac{1}{a} = -\frac{\phi'}{2\phi} + \left[\frac{2\omega + 3}{12} \left(\frac{\phi'}{\phi} \right)^2 + \frac{8\pi a^2}{3\kappa^2 M_P^2 \phi} \left(\rho + \frac{W}{16\pi} \right) \right]^{1/2} \quad , \quad (10)$$

where a prime denotes a derivative with respect to a . Squaring Eq. (10) one obtains

$$W = -16\pi\rho + \frac{\kappa^2 M_P^2}{\phi a^4} [6\phi^2 + 6a\phi\phi' - \omega a^2\phi'^2] \quad . \quad (11)$$

Substituting this expression for W into Eq. (9) yields the following nonlinear ordinary differential equation for ϕ

$$\phi'' = \frac{4\phi^2 + 2a\phi\phi' - a^2\omega\phi'^2}{a^2\phi} + \frac{8\pi a^2}{\kappa^2 M_P^2 (2\omega + 3)} \left[2\frac{\phi}{\phi'}\rho' - 3(\rho + P) \right] \quad . \quad (12)$$

Note that by writing the differential equation in this form we have excluded the case in which the field satisfies the first order equation

$$\frac{1}{a} = -\frac{\phi'}{2\phi} \quad , \quad (13)$$

¹ Notice that this definition of the kinetic and potential energy density for ϕ is somehow imprecise, since their sum does not correspond in general to a covariantly conserved energy density. Nevertheless they are useful quantities to quantify the contribution of the BD field dynamics to the expansion rate and moreover, their sum is covariantly conserved in the limit of very large ω and negligible energy density ρ of standard matter which we discuss in this section, see e.g. [28].

which from Eq. (10) implies that the associated solution is only possible for a negative potential W . Therefore, we shall not consider this case further.

Using the covariant conservation of the energy for standard matter $a\rho' = -3(\rho + P)$, Eq. (12) can be further simplified into

$$\phi'' = \frac{4\phi^2 + 2a\phi\phi' - a^2\omega\phi'^2}{a^2\phi} - \frac{24\pi a(\rho + P)}{\kappa^2 M_P^2 (2\omega + 3)} \left(a + 2\frac{\phi}{\phi'} \right) . \quad (14)$$

We are interested in a situation in which the expansion rate of the universe is of the order of the standard value leading to a successful BBN, and yet the ϕ contribution is relevant or even dominant. By inspection of (6) we see that in order for these conditions to hold, the energy density ρ should be suppressed by a value of $\phi > 1$. Furthermore, the new terms, which are dependent on the dynamics of ϕ , should *mimic* a radiation dominated behavior and compensate for the reduced contribution of ordinary radiation to the expansion rate. In order to see whether this scenario is in fact possible at all, we start by considering the simplest case, in which ordinary radiation and matter contribute negligibly to the kinematics. Of course, this means that we have to check *a posteriori* that, during BBN, it is possible to choose values of ϕ large enough so that ρ is sufficiently suppressed. The leading contribution to (6) will then be provided by the dynamics of ϕ .

If we thus neglect the second term at the r.h.s of Eq. (14), the replacement $s = \phi^{1+\omega}$ linearizes the equation, giving

$$s'' = \frac{2}{a}s' + \frac{4(\omega + 1)}{a^2}s , \quad (15)$$

from which the general solution for $\phi(a)$ can be expressed as

$$\phi(a) = \Phi_0 \left[(1 - C)a^{(3-\Delta)/2} + Ca^{(3+\Delta)/2} \right]^{1/(1+\omega)} , \quad (16)$$

with $\Delta = \sqrt{25 + 16\omega}$, Φ_0 a normalization constant giving the value of the field at $a = 1$, and C determines the initial value of ϕ' via

$$\phi'(1) = \frac{\Phi_0}{2(1+\omega)} [\Delta(2C - 1) + 3] . \quad (17)$$

It should be noticed that in order that the solution remain valid in a neighborhood of the BBN epoch we should have $0 \leq C \leq 1$. Furthermore, it can be checked that the potential W fails to be positive definite if $C > 1$. There are two possible monotonic behaviors for ϕ in the two particular cases $C = 0$ and $C = 1$

$$\phi_{\pm}(a) = \Phi_0 a^{\epsilon_{\pm}} , \quad (18)$$

where $\epsilon_{\pm} \equiv (3 \pm \Delta)/2(1 + \omega) = \pm 8/(\Delta \mp 3)$. The ϵ_+ and ϵ_- indexes correspond to the field rolling towards larger or smaller values respectively, with an extremely slow power-law behavior since, for large ω , we have $|\epsilon_{\pm}| \simeq 2/\sqrt{\omega} < 0.01$. For all other choices of C the solution satisfies a very finely tuned condition corresponding to the field bouncing back at Φ_0 , as shown in Figure 1. In the following therefore, we will only consider the two behaviors described in Eq. (18). Notice that since ω is very large, in the interval $0.01 \leq a \leq 100$ the two solutions can be expressed with an accuracy better than 1% as

$$\phi_{\pm}(a) = \Phi_0 \left(1 \pm \frac{2}{\sqrt{\omega}} \log a \right) . \quad (19)$$

Plugging Eq. (18) into Eq. (11) (and neglecting ρ) one has

$$\frac{W}{\phi} = \kappa^2 M_P^2 (6 + 6\epsilon_{\pm} - \omega\epsilon_{\pm}^2) \left(\frac{\phi}{\Phi_0} \right)^{-4/\epsilon_{\pm}} \simeq 2\kappa^2 M_P^2 \left(\frac{\phi}{\Phi_0} \right)^{\pm 2\sqrt{\omega}} , \quad (20)$$

where the last expression holds for large ω .

Let us summarize the results obtained so far. By considering a potential of the form shown in Eq. (20), the expansion rate of the Universe due to a BD field behaves the same way as during a radiation dominated regime. Depending on the sign of the power-index in W , there are two possible behaviors for the field ϕ , increasing or decreasing with cosmic expansion. The ϕ field rolls very smoothly along the potential, and with comparable “kinetic” and “potential” energy densities K and W/ϕ respectively. Since we derived these results in the limit $W \gg \rho$, we must check that this condition is compatible with the solution found. Indeed, it is easily seen that the term W/ϕ in Eq.(20) is independent from the value of Φ_0 , while the term ρ/ϕ can be made arbitrarily small by choosing Φ_0 arbitrarily large. This implies that provided we choose the initial value for ϕ to be sufficiently large, the contribution of ordinary radiation can be neglected, and the expansion of the Universe follows $a \propto \eta$ despite the fact that it is driven by a BD scalar field.

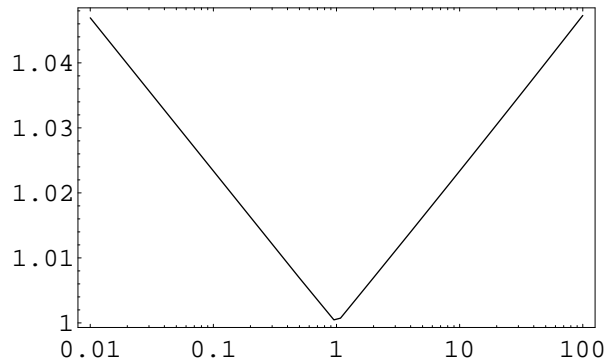


FIG. 1: The evolution of the ϕ/Φ_0 field versus a for $C = 0.5$ and $\omega = 40000$. The field bounces back at $a = 1$ where $\phi = \Phi_0$.

IV. BBN IN A BD-DOMINATED COSMOLOGY

In the previous section we have found that there are two solutions for which a BD field dominates cosmic dynamics, while keeping a radiation-dominated expansion as in Eq. (8). We turn now to the phenomenological requirement of preserving in this scenario the general agreement between BBN predictions and the observed light element yields. Of course, the scenario outlined in the previous section of a pure ϕ -dominated universe cannot describe the late stages of cosmic evolution. Indeed this would be at variance with the fact that today we require $\phi = 1$. Moreover, with the potential considered in the previous section, the universe would continue to expand as in a radiation dominated regime until very small redshifts. We do not address these issues further here, but it is clear that a more complicated dynamics after the BBN period is required in order to bring such kind of models in accordance with “late” cosmological and astrophysical observables. In the following, in particular we will assume that CMB observations are explained as in the standard scenario.

The set of equations governing BBN, besides equations (5) and (6), is the following (see e.g. [55])

$$\frac{1}{n_B} \frac{dn_B}{dt} = -3H \quad (21)$$

$$\frac{d\rho}{dt} = -3H(\rho + P) \quad (22)$$

$$\frac{dX_i}{dt} = \Gamma_i(X_j) \quad (23)$$

$$L(m_e/T, \Phi_e) = \frac{n_B}{T^3} \sum_j Z_j X_j \quad (24)$$

$$(\partial_t - H |\vec{p}| \partial_{|\vec{p}|}) f_{\nu_\alpha}(|\vec{p}|, t) = I_{\nu_\alpha}[f_{\nu_e}, f_{\bar{\nu}_e}, \dots] \quad (25)$$

The first two equations state respectively the conservation of the total baryon number and energy density in a comoving volume. The third equation is the Boltzmann equation which describes the density of each nuclide species, with the Γ_i being the rates of interaction averaged over the kinetic equilibrium distribution functions. The fourth equation accounts for the electric charge neutrality of the universe in terms of the electron chemical potential Φ_e . Finally, the last equation is the Boltzmann equation for the neutrino species.

It is clear that the field ϕ never explicitly enters these equations but does appear in the evolution of the Hubble parameter. Therefore, as long as the evolution of H in the BD case is indistinguishable from the standard case, adopting initial conditions (at whatever $T \gtrsim 1$ MeV) for the matter and radiation fields analogous to the standard case, the production of the light elements will also be unchanged. However, even in the scenario described in the previous section there is one relevant departure, since the Hubble factor evolves strictly as a^{-2} , while in the standard case this evolution is modified because e^\pm annihilation changes the number of effective relativistic degrees of freedom. Note that this phenomenon happens just between the two crucial temperatures for BBN; $T \simeq 0.7$ MeV at which the neutron/proton ratio freezes, and $T \simeq 0.07$ MeV when the deuterium bottleneck opens and light element production begins. If the BD Hubble rate matches one of the two conditions, it might fail to match the second one. To test if such a difference can be accommodated at all, we have run the BBN code described in [55] by assuming that the dominant contribution to H is provided by the BD field, and its behavior is as for pure radiation as discussed in the previous section. To this end we replace the standard

matter/radiation energy density ρ in the Hubble law (and only there!) with the following parametrization in terms of a single neutrino energy density

$$H \rightarrow \sqrt{\frac{8\pi}{3M_P^2}} y \rho_\nu \quad , \quad (26)$$

where $\rho_\nu = 7\pi^2 T_\nu^4/120$ and $T_\nu \propto a^{-1}$. In the limit in which the solution previously found holds, this useful parametrization is *exact*. We ask if there is any value at all for the factor y which can accommodate at least ^4He and ^2H abundances. Since we assume that the standard cosmology holds for $a \gg a_{\text{BBN}}$, we fix the baryon abundance to that deduced from the CMB, Eq. (7). Note that, since there are two conditions to fulfill with only one parameter, the constraint is non-trivial. We find that, for

$$5.5 \lesssim y \lesssim 7.4 \quad , \quad (27)$$

the conservative constraint on the ^4He mass fraction $0.232 \leq Y_p \leq 0.258$ quoted in [58] is satisfied, while predicting a deuterium fraction $1.99 \cdot 10^{-5} \leq ^2\text{H}/\text{H} \leq 2.66 \cdot 10^{-5}$, comparable with the observed one $^2\text{H}/\text{H} = (2.78_{-0.38}^{+0.44}) \cdot 10^{-5}$ [53] within 2σ . For y in the range of Eq. (27) we also found $0.97 \cdot 10^{-5} \leq ^3\text{He}/\text{H} \leq 1.07 \cdot 10^{-10}$, which is consistent with present bounds [59], and $4.2 \cdot 10^{-10} \leq ^7\text{Li}/\text{H} \leq 5.1 \cdot 10^{-10}$, which remains a factor ~ 3 larger than the observed one [60, 61].

In terms of ϕ , this constraint translates into

$$2.35 \sqrt{\frac{8\pi \rho_\nu}{3M_P^2}} \leq -\frac{\dot{\phi}}{2a\phi} + \left[\frac{2\omega + 3}{12} \left(\frac{\dot{\phi}}{a\phi} \right)^2 + \frac{W}{6M_P^2 \phi} \right]^{1/2} \leq 2.72 \sqrt{\frac{8\pi \rho_\nu}{3M_P^2}} \quad , \quad (28)$$

or equivalently

$$5.16 (T_\nu a)^2 \leq \kappa M_P \left(\frac{\epsilon_\pm}{2} + \sqrt{1 + \epsilon_\pm + \frac{\epsilon_\pm^2}{4}} \right) \leq 5.97 (T_\nu a)^2 \quad . \quad (29)$$

The existing bound on ω implies $|\epsilon_\pm| < 0.01$, which in turn simplifies the previous constraint to the following one (with a percent accuracy)

$$5.16 (T_\nu a)^2 \leq \kappa M_P \leq 5.97 (T_\nu a)^2 \quad . \quad (30)$$

This is the constraint on the potential W —or, more precisely, on W/ϕ —of Eq. (20) which accommodates our model with successful nucleosynthesis. From current cosmological measurements one would deduce $T_\nu a \simeq 2\text{ K}$ and constant from the BBN epoch till now while. However, lacking a detailed modelling of the transition from a pure ϕ -dominated phase to a radiation dominated one, we do not elaborate on this bound further².

Since we have shown that BBN is compatible not only with a “minimal” effect, but also with a major role for a BD field, it is reasonable to expect that intermediate cases in which the contribution of ϕ and of ordinary radiation are comparable might be viable. Of course, such scenarios generally involve a high degree of fine tuning in the parameters of the field ϕ and/or in the choice of the potential W . Thus, for illustrative purposes, in the following we shall take a phenomenological approach and discuss the situation where Eq. (6) is replaced by

$$\frac{\dot{a}}{a^2} = H \simeq \left[\frac{8\pi}{3M_P^2} \left(\frac{\rho}{\phi} + \frac{W}{16\pi\phi} + K \right) \right]^{1/2} \rightarrow \left[\frac{8\pi}{3M_P^2} \left(\frac{\rho}{\phi} + y \rho_\nu \right) \right]^{1/2} \quad , \quad (31)$$

and again consider the case for which the BD dynamics corresponds to a pure radiation behavior, but relaxing the hypothesis of a vanishing contribution of ordinary radiation to the Hubble parameter. In the first equality

² In the “standard cosmology”, the value of κ would be

$$\kappa = H a^2 = H_0 a_0^2 \sqrt{\Omega_r},$$

where a_0 and Ω_r are the scale factor and the radiation fraction today including also neutrinos, independently of the fact that at least some of them are non-relativistic today. However, unless one specifies the mechanism for the field ϕ to exit the epoch of ϕ -dominance, one has no way to relate the κ needed at the BBN epoch with present-day cosmological quantities.

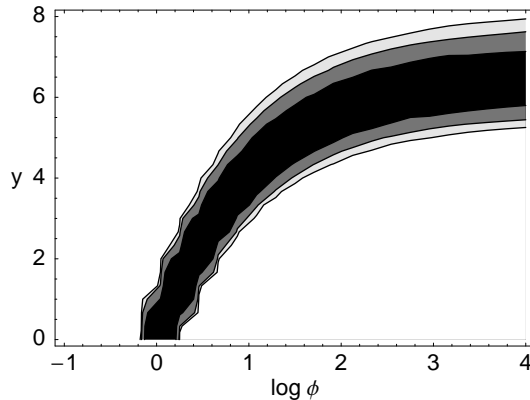


FIG. 2: The 1 (black), 2 (grey) and 3 σ (light grey) contours in the $\phi - y$ plane.

of Eq. (31) we have neglected the term $\dot{\phi}/2a\phi$, which is anyway suppressed by a factor $1/\sqrt{\omega} < 0.01$. Running the modified code for this case leads to the bounds shown in Figure 2, where we plot the likelihood contours in the $\phi - y$ plane at 1, 2 and 3- σ level using both ^2H and ^4He experimental results as before. Two limits are easily recognized. For $\phi \gg 1$ we recover the case considered previously of a dominant BD contribution to H , and correspondingly the bound on y reported in Eq. (27), while for $y \ll 1$, we obtain the “trivial” bound on the effective Newton constant during BBN, namely $0.9 \lesssim \phi \lesssim 1.2$. This result shows that not only both regimes are viable, but also that intermediate situations are phenomenologically allowed, although the details of these cases are more model dependent, and the parameters in the BD sector require a high degree of fine tuning.

It should be noticed that to impose the condition that BD behaves as radiation, or that

$$K + \frac{W}{16\pi\phi} = y\rho_\nu, \quad (32)$$

corresponds to the choice of a particular potential. In principle one should show that this choice is possible. In order to see if this is indeed sensible, we use (32) in equations (5)–(6) and find that a solution can be found by taking the time derivative of these two equations. The outcome is a system of nonlinear 2nd order differential equations in a and ϕ . There is only one independent initial condition—the choice of ϕ_i —since \dot{a}_i can be found by using equation (6), and since equation (5) becomes a cubic equation in $\dot{\phi}$. This means that the system becomes uniquely determined and the solution does indeed exist and it is unique.

V. COMMENTS AND CONCLUSIONS

We have revisited the constraints on a Brans-Dicke scalar field, arising from the requirement of successful nucleosynthesis. This is an important question in light of the different roles that a Brans-Dicke field ϕ may play in various sectors of cosmology, such as inflation, baryogenesis and dark energy.

Indeed, we have found that the BD field, i.e. Newton’s constant, can have significantly different values from today’s one, and at the same time dominate the energy density making the universe behave as if radiation dominates. The matter content is not changed at all, so that all physical quantities such as freeze-out temperatures and decay/production rates for elements do not differ from standard ones.

In other words, we have found that it is possible for the BD field to mimic the effect of radiation on the evolution of the cosmic scale factor. In such a case ϕ is the dominant contributor to the Hubble parameter during BBN. We have put constraints on the BD potential using the observed light elements yields and the BBN predictions in such a modified scenario. This scenario requires a large initial value for $\phi \gtrsim 10$ so that the standard contribution of matter to the expansion rate is suppressed by the reduced value of the effective Newton constant, as well as potential for the BD field W that scales as a power law in ϕ ,

$$W/\phi \propto \phi^{\pm 2\sqrt{\omega}}. \quad (33)$$

Here $\omega \geq 40000$ if we assume that the local result of the Cassini mission also holds on cosmological scales. We do not offer any suggestion how to explain such a high exponent. By numerically solving the set of BBN equations we have found that it is indeed possible to tune the overall scale of the potential W , see Eq. (29), so as to obtain final yields for both ^2H and ^4He compatible with the observed data.

Of course, one may worry that large values of ϕ at the BBN epoch are not compatible with today's value. In fact, if we considered the same effective potential valid until today, the field ϕ would keep on being the dominant component of the universe, leading to a completely different evolution for the universe, e.g. an extrapolation would lead to value for the field ϕ incompatible with today's value for the Newton gravitational constant. Therefore, after BBN, the potential should be chosen such that the field decreases to acceptable values, and the universe follows a standard evolution. The fact that today's value of ϕ imposes an upper bound on the possibility of having very large values for ϕ_{BBN} is tantamount to saying that a fine-tuning of the choice of the potential is necessary in order to allow a huge change in the value of the field from BBN up to now.

Finally, we have considered the more general scenario where both standard radiation and the BD field contribute to the expansion rate, in the case that the BD dynamics again corresponds to a pure radiation behavior. In this case too, by comparing the theoretical prediction with light nuclei abundances, we have found that, depending on the value of the BD field during BBN and its contribution to H , there is a region in this two-parameter space leading to successful nucleosynthesis. This region shown in Figure 2 extends from the high ϕ regime already discussed down to the standard scenario with $\phi = 1$ showing that provided the ϕ potential is suitably tuned, basically all values of the BD field can be shown to be compatible with the abundances of both ^2H and ^4He produced during BBN.

An interesting feature of the scenario considered in this paper is that one is basically free to adjust the time-temperature relation, which analytically means rescaling the initial conditions for the thermodynamical quantities of the fluids in the plasma. We have limited our discussion to the case where $H(a)$ around the BBN epoch has the same evolution it would have in a radiation-dominated plasma, keeping the particle densities fixed at the same value of the standard case as our initial conditions. This is not a fine-tuning required a priori, but an useful working hypothesis which automatically ensures that the rates for the scattering processes (including the nuclear reactions) are unchanged. However, one may relax this assumption, and thus the allowed region in the model space may enlarge considerably, to the price of major (and model-dependent) modifications in the physical processes in the plasma. We do not discuss further this point in our phenomenological approach, since our purpose in this article was to show that exotic scenarios where the BD field is driving the expansion rate at the BBN epoch are indeed possible. Nonetheless, this feature should be kept in mind when characterizing specific models (as e.g. quintessence potentials with solutions tracking radiation) with a cosmic evolution before and after the BBN epoch determined self-consistently within the theory.

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